Theorem: for all n, $2 + 2^2 + 2^3 + ... + 2^n = 2^{(n+1)} - 2$

Proof: By induction. The first case is clear: $2 = 2^2 - 2 = 2$ so the left and right sides are equal. We will assume the nth case as our Induction Hypothesis: $2 + 2^2 + 2^3 + ... + 2^n = 2^{(n+1)} - 2$. Next we will show that assuming the nth case implies the n+1 case: $2 + 2^2 + 2^3 + ... + 2^n + 2^{(n+1)} = 2^{(n+2)} - 2$.

Begin with the Induction Hypothesis: $2 + 2^2 + 2^3 + ... + 2^n = 2^{(n+1)} - 2$

Add $2^{(n+1)}$ to both sides: $2 + 2^2 + 2^3 + ... + 2^n + 2^{(n+1)} = 2^{(n+1)} - 2 + 2^{(n+1)}$

Rearranging the right side: $2 + 2^2 + 2^3 + ... + 2^n + 2^{(n+1)} = 2^{(n+2)} - 2$

This is the n+1 case for the Induction Hyopthesis and so the Induction step is complete. This proves the theorem.